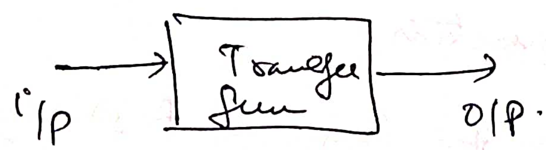
Block diagrams

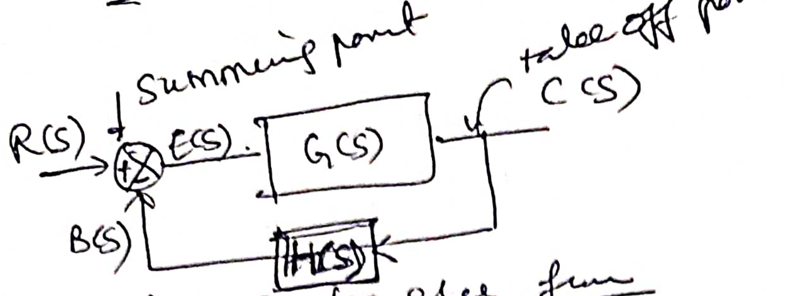
A block diagram of a system is a pictorial representation of the function performed by each component ~~&~~ of the flow of signals. Such a diagram depicts the interrelationships that exists among the various components.

In a block diagram all system variables are linked to each other through functional blocks. Block is a symbol for mathematical operations on the input signal to the block that produces the output. The transfer fun of the components are usually entered in the corresponding blocks which are connected by arrows to indicate the direction of flow of signals

Eg



Block diagram of closed loop sys



closed loop transfer fun

$$C(S) = G(S)\,E(S)$$

$$E(S) = R(S) - B(S)$$

$$C(S) = G(S)\left[R(S) - B(S)\right]$$

$$C(S) = G(S)\left[R(S) - \cancel{H}H(S)\,C(S)\right]$$

$$C(S) + H(S)\,C(S) = G(S)\,R(S)$$

$$C(S)\left(1 + G(S)H(S)\right) = G(S)\,R(S)$$

$$\boxed{\frac{C(S)}{R(S)} = \frac{G(S)}{1 + G(S)\,H(S)}}$$

or

$$\boxed{\frac{C(S)}{R(S)} = TF = \frac{G}{1 + GH}}$$

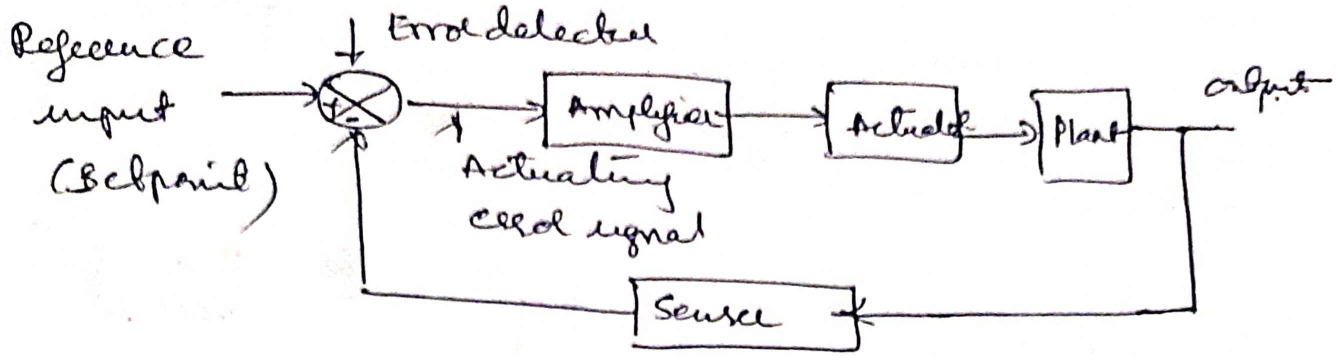
Open loop transfer function

$$\text{Open loop transfer fun} = \frac{B(S)}{E(S)} = G(S)\,H(S)$$

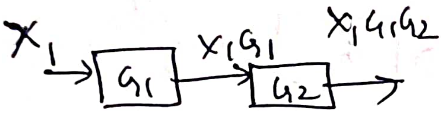
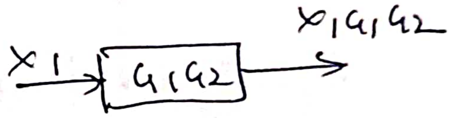
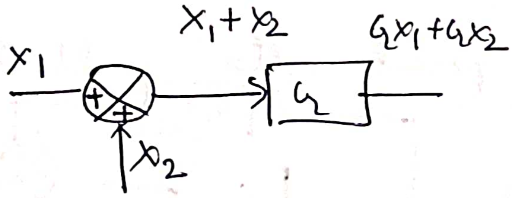
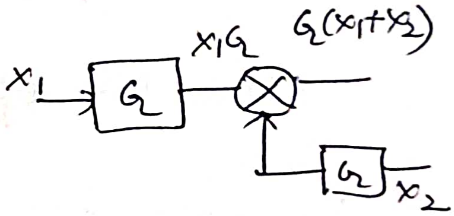
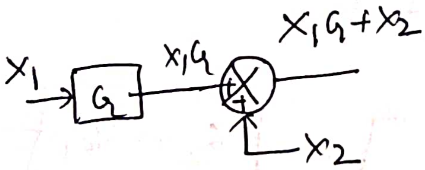
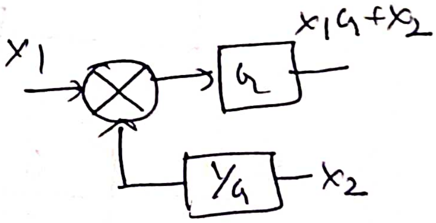
$$\frac{H(S)\,C(S)}{R(S) - B(S)} = \frac{H(S)\,C(S)}{\cancel{R(S)}\;E(S)} = \frac{H(S)\cancel{E(S)}G(S)}{\cancel{E(S)}}$$

B(S) = feedback signal   E(S) actuating error signal

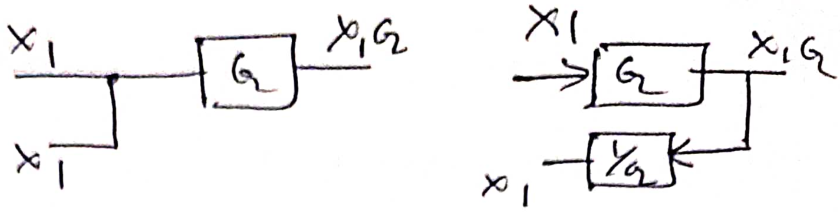
Block diagram of an industrial control system is as follows.



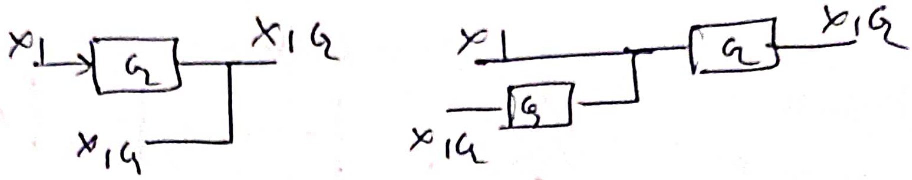
Rules of Block diagram algebra.

Rule	Original diagram	Equivalent diagram
1) Combining blocks in cascade		
2) Moving summing point after a block		
3) Moving summing point ahead of block		

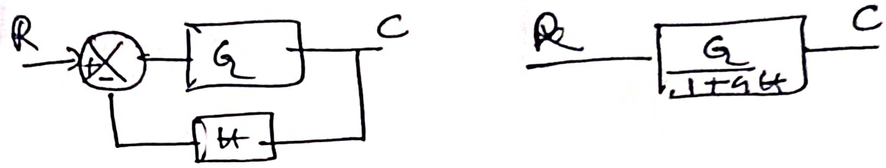
4) Moving a take off point after a block



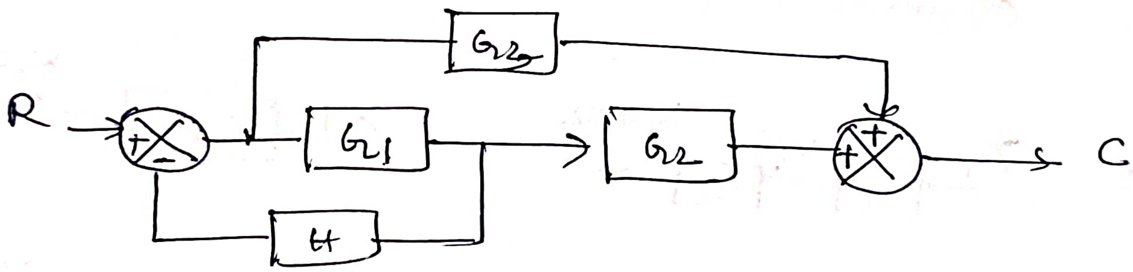
5) Moving a take off point ahead of block



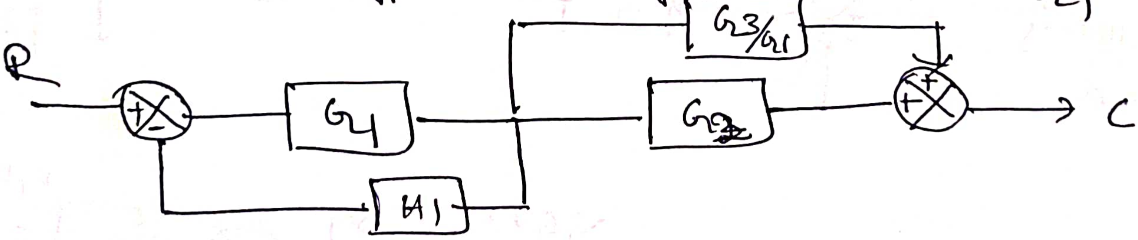
6) Eliminating a feedback loop

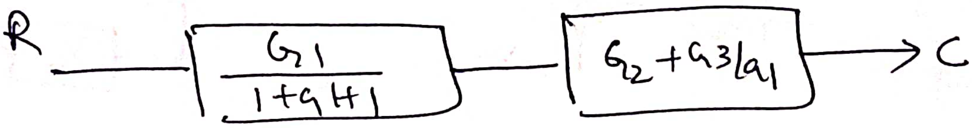


Eg. Reduce the block diagram shown & find C/R



Move take off point after a block G_1

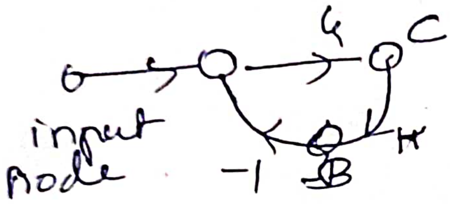




$$\therefore C/R = \frac{G_1\,(G_2+G_3/G_1)}{1+G_1H_1} \qquad \frac{G_1G_2+G_3}{1+G_1H_1}$$

Signal flow graph

Block diagrams are very successful for represent-ing control systems but for complicated systems the block diagram reduction process is tedious & time consuming. An alternate approach is that of signal flow graphs developed by S.J Mason which does not require any reduction process because of availability of a flow graph gain formula which relates input & output system variables.



Mason's gain formula

The relationship between an input variable and an output variable of a signal flow graph is given by the net gain between the input and output nodes and is known as overall gain of the system.

$$T = \frac{1}{\Delta} \sum_k P_k \Delta_k$$

P_k = path gain of k^{th} forward path

Δ = determinant of the graph

= 1 − (sum of loop gains of individual loops)

+ (sum of gain product of all possible combination of two non touching loops) − ...

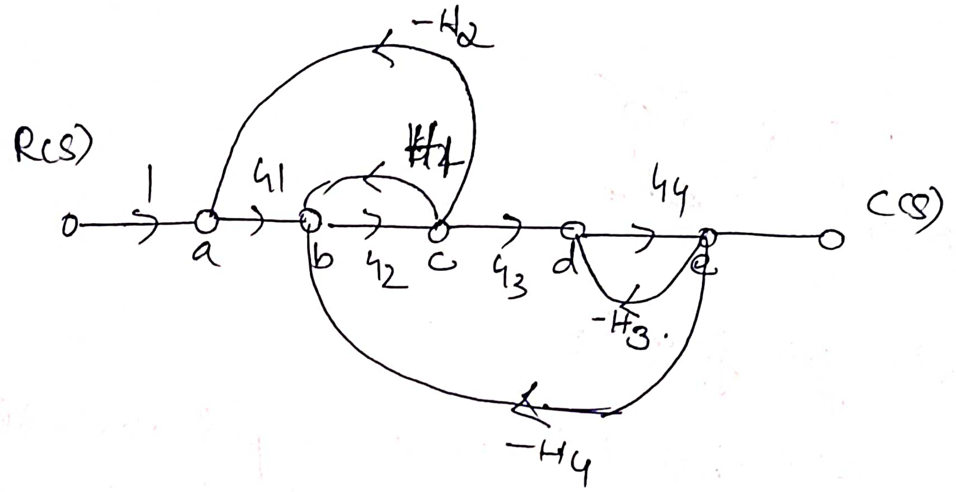
$$\Delta = 1 - \sum_m P_{m1} + \sum_m P_{m2} - \sum_m P_{m3} + - -$$

P_{mr} = gain product with possible combination of r nontouching loops

Δ_k = the value of Δ for the part of graph not touching the kth forward path

T = overall gain of the sys.

<u>Determine</u> C/R using Mason's gain formula



<u>forward path</u>

$P_1 = G_1 G_2 G_3 G_4$

loops L_1 - a-b c-a $= -G_1G_2H_2$

L_2 - b-c-b $= -G_2H_1$

$L_3 \rightarrow$ d-e d $= -G_4H_3$

$L_4 \rightarrow$ b-c-d-e-b $= -G_2G_3G_4H_4$

<u>nontouching loops</u>

$(-G_4H_3)(-G_2H_1) = G_2G_4H_1H_3$

$-G_4H_3(-G_1G_2H_2) = G_1G_2G_4H_2H_3$

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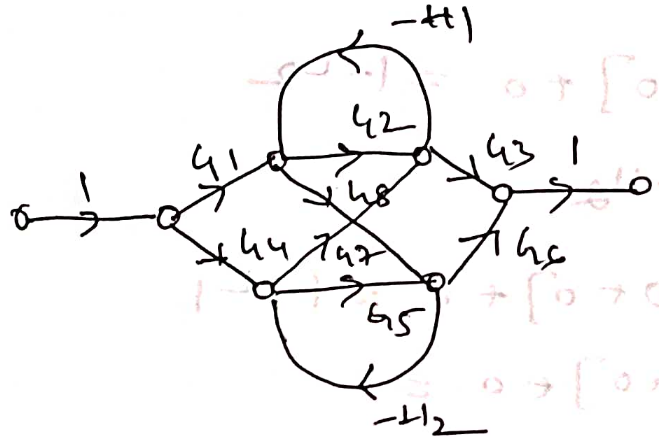
$$T = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 + P_4\Delta_4}{\Delta}$$

$$T = \frac{G_1G_2G_3G_4}{1 + G_1G_2H_2 + G_2H_1 + G_4H_3 + G_2G_3G_4H_4 + G_1G_2G_4H_2H_3}$$

Find C/R for the graph shown below using Mason's gain formula



$$T = \frac{\Sigma \Delta_K P_K}{\Delta}$$

① Forward path with gain

$P_1 = G_1G_2G_3$
$P_2 = G_4G_5G_6$
$P_3 = G_1G_8G_6$
$P_4 = G_4G_7G_6$
$P_5 = G_1G_8 - H_2 G_7G_3$
$P_6 = G_4G_6 - H_1 G_8G_6.$

Individual loops

$L_1 = -G_2H_1 \qquad L_2 = -G_5H_2$

$L_3 = -H_1G_8 \; -H_2G_7$

$= G_7G_8H_1H_2$

Two non-touching loops

$L_1L_2 = G_2H_1G_5H_2$

$\Delta = 1 - [L_1+L_2+L_3] + L_1L_2$

$\Delta_1 = 1 - [0+L_2+0]+0 = 1-L_2$

$= 1+G_5H_2$

$\Delta_2 = 1-[L_1+0+0]+0 = 1-L_1$

$\Delta_3 = 1-[0+0+0]+0 = 1$

$\Delta_4 = 1$

$\Delta_5 = 1$

$\Delta_6 = 1$

$$\frac{C(S)}{R(S)} = \frac{P_1\Delta_1+P_2\Delta_2+P_3\Delta_3+P_4\Delta_4+P_5\Delta_5+P_6\Delta_6}{\Delta}$$

$$= \frac{G_1G_2G_3(1+G_5H_2)+G_4G_5G_6(1+G_2H_1)+G_1G_8G_6 + G_4G_7G_3 - G_1G_8H_2G_7G_3 - G_4G_7H_1G_8G_6}{\Delta}$$

$\Delta = 1+G_2H_1+G_5H_2 - H_1G_8H_2G_7 + G_2H_1G_5H_2$

$$\therefore \; TF = \frac{G_1G_2G_3(1+G_5H_2)+G_4G_5G_6(1+G_2H_1)+G_1G_8G_6 + G_4G_7G_3 - G_1G_8H_2G_7G_3 - G_4G_7H_1G_8G_6}{1+G_2H_1+G_5H_2-H_1G_8H_2G_7+G_2H_1G_5H_2}$$

Derive transfer function of a dc servomotor

In servo applications DC motor is required to produce rapid accelerations from standstill.

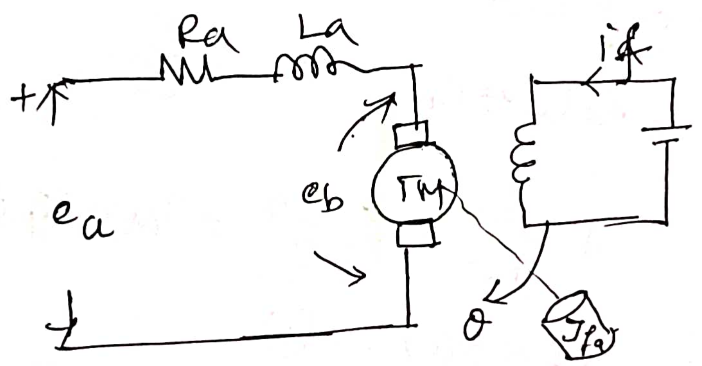
∴ physical requirement of such motor are low inertia and high starting torque.

Armature control

Consider the armature controlled DC motor

R_a = resistance of armature (Ω)

L_a inductance of armature winding (H)



i_a = armature current

i_f = field current

e_a = applied armature voltage

e_b = back emf

T_M = torque developed by motor shaft

J = Equivalent moment of inertia

f_0 = viscous friction coefficient of motor & load to motor shaft (Nm/rad/s) air gap flux

$$\phi = k_f i_f$$

$$\phi \propto i_f \text{ (field current)}$$

k_f constant

$$T_M = k_1 k_f i_f i_a$$

k_1 is a const

In armature controlled dc motor the field current is kept const.

$$T_M = k_T i_a$$

$$e_b = k_b \frac{d\theta}{dt} \quad \text{— (a)}$$

back emf is proportional to speed

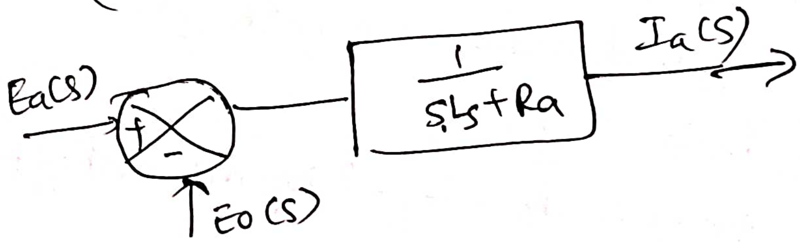
k_b - back emf const.

$$L_a \frac{di_a}{dt} + R_a i_a + e_b = e_a \quad \text{— (1)}$$

Torque eqn $\quad J \frac{d^2\theta}{dt^2} + f_0 \frac{d\theta}{dt} = T_M = K_1 i_a$

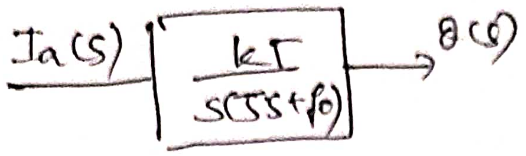
$$E_b(s) = k_b s\, \theta(s) \quad \text{— (b)}$$

$$(L_a s + R_a) I_a(s) = E_a(s) - E_b(s) \quad \text{— (2)}$$



$$(Js^2 + f_o s)\,\theta(s) = T_M(s) = k_T I_a(s)$$

$$\theta(s) = \frac{K_T}{s(Js+f_o)}\, I_a(s)$$



T. F of the sys.

$$G(s) = \frac{\theta(s)}{E_a(s)} = \frac{k_T}{s\left[(R_a + sL_a)(Js+f_o) + k_T k_b\right]}$$

$$E_b(s) = k_b\, s\, \theta(s)$$

$\cancel{\theta(s)}$. E